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TITLE A SOLUTION TO NONLINEARITY PROBLEMS

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A SOLUTION TO NONLINEARITY PROBLEMS*

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Abstract New methods of correcting dynamic nonlinearities resulting from the multipole content of a synchrotron or transport line are presented. In a simplest form, correction elements are placed at the center (C) of the accelerator half-cells as well as near the focusing (F) and defocusing (D) quadrupoles. In a first approximation, the corrector strengths follow Simpson's Rule, forming an accurate quasi-local cancelling approximation to the nonlinearity. The F, C, and D correctors may also be used to obtain precise control of the horizontal, coupled, and vertical motion. Correction by three or more orders of magnitude can be obtained, and simple solutions to a fundamental problem in beam transport have been obtained.

INTRODUCTION

Future synchrotrons will use high-field conductor dominated superconducting dipoles; these magnets have relatively large nonlinear (multipole) fields from persistent current, conductor placement, and saturation effects. The greatly increased circumferences of the highest energy machines magnify the nonlinear effects, while forcing the designs toward smaller aperture, more nonlinear magnets. Beam stability demands highly linear motion and, therefore, linear fields. In the Superconducting Super Collider (SSC), linear motion tolerances for multipole content are at the 10^{-6} cm⁻² level whereas expected strengths of the lower multipoles are near or above the 10^{-4} level. Correction of b_2 , b_4 , and b_6 (sextupole, octupole, 10 pole) is necessary.¹

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Previously, synchrotron dynamics was dominated by dipole, quadrupole, and first-order sextupole effects, and corrector elements near focusing (F) and defocusing (D) quads were adequate. However, correctors near the quads are completely ineffective for higher orders. Before the discoveries described in this paper, it was believed necessary to include internal b_2 , b_3 , and b_4 trim coils along the length of every dipole for local cancellation of nonlinearities.² However, such internal coils greatly complicate the dipoles and are impractical.

In May 1987, the author considered the possibility of including correctors in the center (C) of accelerator half-cells (see Fig. 1) and immediately discovered enormous improvements, including the elimination of any need for internal trim coils.³ In further elaborations, the author and his collaborators have found that the methods are much more general and powerful than the initial evaluations and have firmly connected them with basic physical principles.⁴⁻¹⁰

PHYSICAL BASES OF THE CORRECTION

Figure 1 shows the correction method as applied in its simplest form in a symmetrical FODO transport cell, with correctors in the center (C) of the half cell as well as near the F and D quads. On the half-cell level, the correctors form a three-point (F, C, D) system. Application of basic physical principles to this system provides extremely accurate compensation and control of all nonlinearities

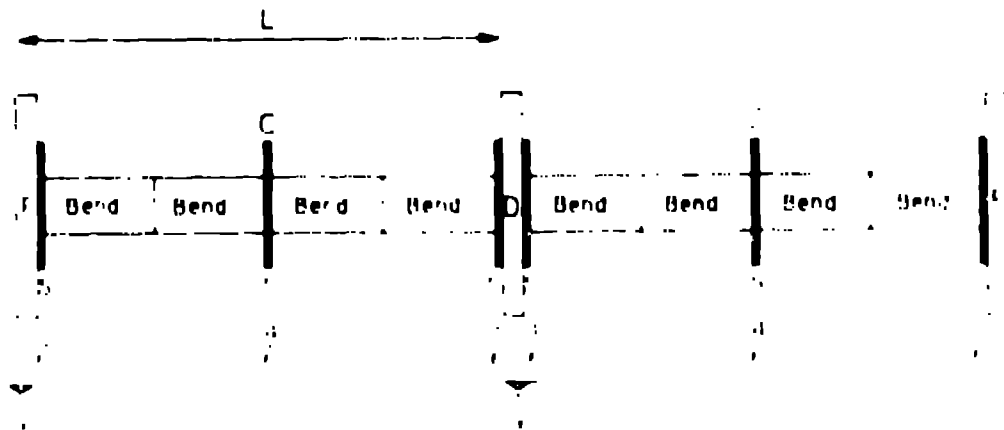


Fig. 1. A sample collider cell. The element labels are F and D = quads, S = corrector in the center, and C = correction strengths on opposite sides of the thin quads. (When the quads are on other sides, there are only two correctors per half cell.)

The (F, C, D) correctors can be adjusted to form an optimal, quasi-local cancellation of the continuous multipole content of the dipoles.⁶ The magnetic fields in the dipoles may be expressed as

$$B_y + iB_x = B_0[1 + \Sigma(b_n + ia_n)(x + iy)^n] \quad ,$$

where b_n and a_n are the normal and skew multipole strengths. For the case of constant (systematic) multipole content, the optimum corrector strengths S_i are close to Simpson's Rule values: $(S_F, S_C, S_D) = (1/6, 4/6, 1/6)B_0b_nL$. That solution reduces all nonlinear effects by two or more orders of magnitude. This indicates that the (F, C, D) correctors are fully equivalent to the continuous distribution at the 1% level. A similar algorithm has been developed to compensate varying (random) multipole content; similar cancellations are obtainable.⁵⁻¹⁰

The (F, C, D) correctors are also at optimal locations for separated-function control of horizontal-, coupled-, and vertical-motion parameters, and these are precisely the operational observables. This tunability can be used in improving correction from initial approximations. For instance, (F, C, D) octupoles are appropriate elements for control of all amplitude-dependent and second order chromatic tune shifts.⁴⁻⁷ The (F, C, D) elements permit exact control of the motion through 10 pole order.

EXAMPLES OF CALCULATIONS

Table I displays calculations of b_1 and b_4 tune spreads and linearity tolerances in a large collider lattice. As discussed above, $b_1, b_4 \lesssim 10^{-6} \text{ cm}^{-1}$ is desired without correction. Adding correctors only near the F and D quads is completely ineffective in improving linearity. Correctors at only C locations are slightly more effective. However, (F, C, D) correctors reduce all Δx terms by $10^2 - 10^4$, improving tolerances to $\sim 10^{-4} \text{ cm}^{-1}$.

TABLE I. Correction in a sample large collider lattice, calculated with $b_n = 10^{-4} \text{ cm}^{-n}$ in a lattice of cells with $L = 100 \text{ m}$, $\theta = 0.5^\circ$, and $\phi = 90^\circ$. Tolerances require that the tune spread over all trajectories with $A_x, A_y < 0.5 \text{ cm}$ and $\Delta p/p < \pm 0.001$ be less than 0.005. The correction factor is the ratio of uncorrected to corrected tune spreads.

Correction Conditions	Octupole (b_3) Correction		Tolerance 10^{-4} cm^{-n}
	Tune Spread	Correction Factor	
No correction	0.301	1.0	0.016
F, D only ($f_F=0.5, f_D=0.5$)	0.137	2.2	0.036
F, C, D Correction, ($f_F = f_D = 1/6, f_C = 2/3$)	0.0041	73.1	1.2
($f_F = f_D = 0.1647$, $f_C = 0.6571$)	0.0006	500.0	8.3
	Decupole (b_4) Correction		
	Tune Spread	Correction Factor	
No correction	0.29	1.0	0.025
F, D only ($f_F=0.5, f_D = 0.5$)	0.20	1.0	0.025
F, C, D Correction ($f_F = f_D = 1/6, f_C = 2/3$)	0.0066	30.0	0.76
($f_F = 0.1588, f_D = 0.1686$, $f_C = 0.6614$)	0.0001	2000.0	50.000

The accuracy of the correction can be understood by noting that any $\Delta\nu$ term can be expressed as an integral over the lattice. For example, a b_3 term may be written as

$$\Delta\nu_3 = \int b_3 J_x^3 ds = S_{1,F} J_x(0)^2 + S_{1,C} J_x(L/2)^2 + S_{1,D} J_x(L)^2. \quad (1)$$

All other nonlinearities can be expressed as similar integrals. The correction is equivalent to approximating a continuous integration by a sum over discrete points. Simpson's Rule is a generally valid solution; it reduces all nonlinearities by $\approx 10^2$. Optimization about that solution can reduce critical nonlinearities by another order of magnitude (see Table I.)

Other nonlinear effects, such as orbit distortion and higher order $\Delta\nu$, are also reduced by large factors. For instance, the Collins b_2 distortion functions¹¹ are exactly cancelled to zero at the half cell level by Simpson's Rule correctors. Forest and Peterson⁷ have extended the method to correct random multipole content. The F, C, and D corrector strengths are set by requiring that the lowest order moments of the multipole content plus the correctors be zero on the half cell level. Merminga and Ng¹⁰ have obtained calculations that show large reductions in orbit distortions from systematic and random multipole content.

The (F, C, D) octupoles are appropriate elements for separated-function control of the horizontal, coupled, and vertical amplitude-dependent tune shifts,^{4,7} whether these terms come from octupole content, second-order sextupole, or interaction region (IR) effects. The F and C (not D) octupoles can also control second-order chromaticity. Correction by several orders of magnitude is possible, and b_2 tolerances can be increased to $\approx 10^{-2} \text{ cm}^{-2}$.

VARIATIONS, EXTENSIONS, AND APPLICATIONS

Variations on corrector configurations have also been studied.⁸ One interesting variation places two correctors in each half-cell at the locations for Gaussian Quadrature.⁶ It shows similar $\Delta\nu$ correction, but inferior tunability to the (F, C, D) correction.

The strong focusing and large betatron functions in IR quads magnify their nonlinearities. The present methods can provide accurate correction of IR nonlinearities either quasi-locally within the IR magnet array or remotely using arc correctors.

Although initial evaluations were of the SSC, the same methods can and should be applied to any transport, with similar improvements. A key ingredient, which should be applied generally, is the inclusion of correctors at the half-cell centers (where $J_x \cong J_y$) for the coupled motion.

An initial evaluation showing accurate correction for the Large Hadron Collider (LHC) has been obtained; the correction is adequate enough to permit weaker focusing and therefore more dipole length, even after allotting space for correctors.⁷

The HERA design was completed before the discovery of the present methods; it includes many families of trim coils with nonoptimal placements.¹² Modifications would be desirable. The Relativistic Heavy Ion Collider (RHIC) 1986 design¹¹ unfortunately contained correctors only near the quads, including octupoles and 10 poles. With that placement, those elements are worthless. The design should be modified to include some corrector slots near the half cell centers.

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